Quantum Gravity

§ Why ?

gravitational wave detected !

→ Need to quantize the theory of gravity in order to avoid the ultra-violet catastrophe.

§ Path-integral formulation

$$\int \underbrace{\mathcal{D} g_{\mu\nu}(\sigma)}_{\text{metric}} \; e^{-S_E[g_{\mu\nu}]}$$

Euclidean two-dim'l "pure" gravity.

$$S_E[g_{\mu\nu}] = \frac{1}{4\pi}\int d^2\sigma \sqrt{g} \cdot \Big[\Phi_0 R + \Lambda \Big]$$

curvature

Φ_0: const. $\sim \begin{bmatrix}\text{Newton} \\ \text{const}\end{bmatrix}^{-1}$

Λ: cosmological const.

① note that

$$\frac{1}{4\pi}\int d^2\sigma \sqrt{g} \cdot R = \chi(M)$$

"topological"

Euler charateristic !

$$g_s = e^{\Phi_0}$$

$$\int Dg_{\mu\nu}\, e^{-S_E[g_{\mu\nu}]} = \sum_{h} g_s^{-\chi(M_h)} \cdot \underbrace{\int [Dg_{\mu\nu}]_h \left(e^{-\frac{\Lambda}{4\pi}}\right)^{\int d^2\sigma \sqrt{g_h}}}$$

h: number of genus

M_h

$$\chi(M_h) = 2(1-h)$$

$$\int d^2\sigma \sqrt{g_h} = \mathrm{Area}(g^h_{\mu\nu})$$

$$= \sum_h g_s^{-\chi(M_h)} \underbrace{\int [Dg_{\mu\nu}]_h \left(e^{-\frac{\Lambda}{4\pi}}\right)^{\mathrm{Area}(g_{\mu\nu})}}$$

top'l ~~sum~~ expansion

$\boxed{Q}$: how to define this ?

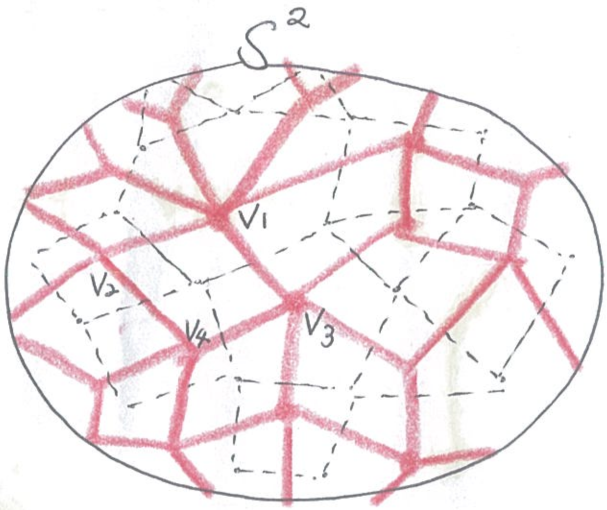
A: continuum approach

discretization of 

followed by the continuum limit

let's take this approach !

§ Discretization of surface



quadrangle lattice

(quadrangulation)

area of each quadrangle

$$= \sum_{\substack{\text{all}\\ \text{quadrangulation}}} \left(e^{-\frac{\Lambda}{4\pi} \blacksquare} \right)^{V}$$

number of quadrangles

$\cong$ # of vertices of <u>dual</u> lattice

"<u>random matrix model</u>"

$\Uparrow$ ~~continuum limit~~

Use

① Feynman diagram & large N

$$\int [DM]\, e^{-V(M)}$$

② Saddle-point approximation

$N \times N$ hermitian matrix

③ orthogonal polynomial method

§ continuum limit ?

$$\left.\begin{array}{l} V \to \infty \\ \text{▨} \to 0 \\ \text{s.t. } \; V\,\text{▨} \sim \text{const} \end{array}\right\} = \text{double scaling limit of matrix model}$$

[<u>remark</u> ~ rare-event analysis of statistical system]

§ gravity + matter

$$S_E[g_{\mu\nu}(\sigma), X(\sigma)] = \frac{1}{4\pi}\int d^2\sigma \sqrt{g}\,\Big\{ \Phi_0 R + \Lambda + \underbrace{\frac{1}{\alpha'} g^{\mu\nu}\partial_\mu X \partial_\nu X}_{\text{Polyakov string}} \Big\}$$

$(\alpha' = 1)$

X: non-compact scalar field

(1-dim'l embedding of 2-d string world-sheet)

Performing the quadrangulation of the surface,

$$Z(\Phi_0, \Lambda) = \sum_{g} (e^{\Phi_0})^{2g-2} \sum_{\underset{\text{lattice}}{L}} \Big(e^{-\frac{\Lambda}{4\pi} \underset{\text{unit area}}{a^2}} \Big)^{\overset{\#\text{ of vertices of dual lattice } \tilde{L}}{V}} \times \underbrace{\int_{-\infty}^{\infty} \prod_{i=1}^{V} dX_i}_{\prod_{\langle i,j\rangle} e^{-|X_i - X_j|^2}}$$

<u>remarks</u>

$$\underbrace{\int_{-\infty}^{\infty} \prod_{i=1}^{V} dX_i \underbrace{\prod_{\langle i,j\rangle} e^{-|X_i - X_j|^2}}_{f(x_i - x_j) \text{ imples the translational sym.}}}_{\text{thus this integral becomes divergent } \propto (\text{vol. of } X)}$$

remarks

$$\frac{1}{4\pi}\int d^2\sigma \sqrt{g}\left\{ g^{\mu\nu}\partial_\mu X \partial_\nu X \right\} \sim \sum_{\langle i,j\rangle}' (X_i - X_j)^2$$

sum over all links

bet'n vertices in $\tilde{\Sigma}$

it was argued that there should exist a whole universality class of link factors $f(|X_i - X_j|)$ that result in the same continuum theory

$$\underbrace{\begin{aligned} f_1(x,y) &= e^{-(x-y)^2} \\ f_2(x,y) &= e^{-|x-y|} \\ &\vdots \end{aligned}}_{\text{the same universality class.}}$$